5

Optimal Cascades Profiling

There are two different approaches to determining the optimal parameters of planar cascades of profiles for the designed axial turbine flow path.

The first one which is suitable for the early stages of design, does not takes into account the real profile shape, i.e. based on the involvement of empirical data on loss ratio, geometrical and strength characteristics depending on the most important dimensionless criteria (the relative height and pitch, geometric entry and exit angles, Mach and Reynolds numbers, relative roughness, etc.). The advantages of this approach are shown in the calculation of the optimal parameters of stages or groups of stages, as allow fairly quickly and accurately assess the mutual communication by various factors – aerodynamics, strength, technological and other, affecting the appearance of created design – and make an informed decision.

The second approach involves a rigorous solution of the profile contour optimal shape determining problem on the basis of a viscous compressible fluid flow modeling with varying impermeability boundary conditions of the profile walls. In practice, the task is divided into a number of sub-problems (building the profile of a certain class curve segments, the calculation of cascade fluid flow, the calculation of the boundary layer and the energy loss) solved repeatedly in accordance with the used optimization algorithm, designed to search for the profile configuration that provides an extremum of selected quality criteria (e.g., loss factor) with constraints related to strength, and other technological factors.

5.1. The Cascade’s Basic Geometry Parameters Optimization

The importance of solving the problem of the cascade’s basic characteristics definition can be seen from the following considerations. Let designed axial turbine stage blades at a predetermined height. Under certain parameters before and behind the stage is usually determined the number of blades and profile chords so that with an energy loss minimum satisfy strength and vibration requirements. The simplest solution is to select the "optimal" t/b ratio using known empirical relationships and determining the chords provide reliable operation. Upon closer examination the situation is not so simple: first, the optimum ratio t/b is determined by many factors (the relative thickness of the edge, the Reynolds number and the relative roughness of the surface, relative height and others); secondly, the permissible loss and the vibration characteristics depend on the influence of the previous cascade; third, the stage design can be carried out both from the set of standard profiles or suggest subsequent entirely new cascades profiling. Consideration of these circumstances makes the task of optimization of the basic cascade parameters quite challenging and promising in terms of using hidden in complicated situations reserves to increase efficiency and reduce consumption of materials in the created turbomachine design.

The calculation of the kinetic energy loss on the basis of empirical relationships has repeatedly been considered and, as experience shows, in the form set out in Chapter 2, is a reliable tool to assess the various components of the losses in the cascade. Calculation of the geometric characteristics of the profiles is carried out using a dependency suitable for working and nozzle profiles, including an elongated front portion. The stresses in the diaphragms, nozzle and rotor blades, as well as restrictions on the vibrational reliability calculated by the well-known and, as far as possible, the exact dependence.

When optimizing an isolated cascade the following problem statements species are considered.

I. Profile presentation method

I.1. Standard profile. The geometric characteristics are determined by the tabular data and restated for a specific profile stagger in the cascade, which provides the desired output stream angle at a known relative pitch.

I.2. "Macromodel". The form of the profile is not known beforehand, but its defining geometrical characteristics can be estimated by empirical dependence of the type [26].

I.3. Profiling. In addition to the previous statement can be built demo profile, designed by a faster way. It is possible geometrical and strength characteristics evaluation on its configuration.

II. Variable parameters.

II.1. Optimization of chord when t/b = const.

II.2. Optimization of t/b when b = const.

II.3. The chord and the relative pitch optimization. In constructing the cascade of the standard profiles the profiles chord selection is in sequential enumeration of profiles of this type, but of different size [20, 33].

III. Boundary conditions.

III.1. Geometric, kinematic and gas-dynamic parameters in the first approximation are given from stage thermal calculation.

III.2. Cascade optimization process is conducted directly to the stage (multistage flow path) thermal calculation and optimization. In this case, the design of the cascade is embedded in an iterative process instead of the verifying energy losses in cascades, as is usually done.

Optimization is made by LP- search, and where this is not possible, brute force at defined ranges of variable parameters and the number of sampling points. The calculation is carried out in designer's dialogue with a computer, which significantly reduces the time to find an acceptable solution.

5.2 Profiles Cascades Shaping Methods

The resulting thermal calculations of optimal geometry and gas-dynamic parameters of the working fluid at the inlet and outlet of the blade row let you go to the next stage of optimization of the turbine flow path – the blade design. The solution of the latter problem, in turn, can be divided into two stages: the creation of planar profiles cascades and their reciprocal linkage also known as stacking [25].

The optimal profiling problem formulated as follows: to design optimal from the standpoint of minimum aerodynamic losses profiles cascade with desired geometrical characteristics, provides necessary outlet flow parameters and satisfying the requirements of strength and processability.

To optimize the cascade's profile shape profiling algorithm is needed, satisfying contradictory requirements of performance, reliability, clarity and high profiles quality.

Earlier, considerable effort has been expended to develop such algorithms [25]. Analyzing the results of these studies, the following conclusions may be done. First, great importance is the right choice of a class of basic curves, of which profiles build (which may be straight line segments and arcs, lemniscate, power polynomial, Bezier curves, etc.), which primarily determines the reliability and visibility of solutions. The quality of the obtained profiles associated with the favorable course of the curvature along the contours, the choice of which is carried out using the criteria of "dominant curvature", minimum of maximum curvature, and other techniques.

First, consider the method of profiles constructing with power polynomials [15, 34]. The presentation will be carried out in relation to the rotor blade.

5.2.1 Turbine Profiles Building Using Power Polynomials

Initial data for the profile construction. Analysis of the thermal calculation results (entry β_1 and exit β_2 angles, values of flow velocities W_1 and W_2), and the requirements of durability and processability lead to the following initial profiling data (Fig. 5.1): β_{1g} – constructive entry angle; f – cross-sectional area; b – chord; t – cascade pitch. Optimal relative pitch of the cascade can be determined beforehand on the recommendations discussed in [25]; a – inter-blade channel throat; ω_1 – entry wedge angle; r_1 – the radius of the leading edge rounding; r_2 – the radius of the trailing edge rounding; ω_2 – exit wedge angle; β_s – profile stagger angle; β_{2g} – constructive exit angle; δ – unguided turning angle.

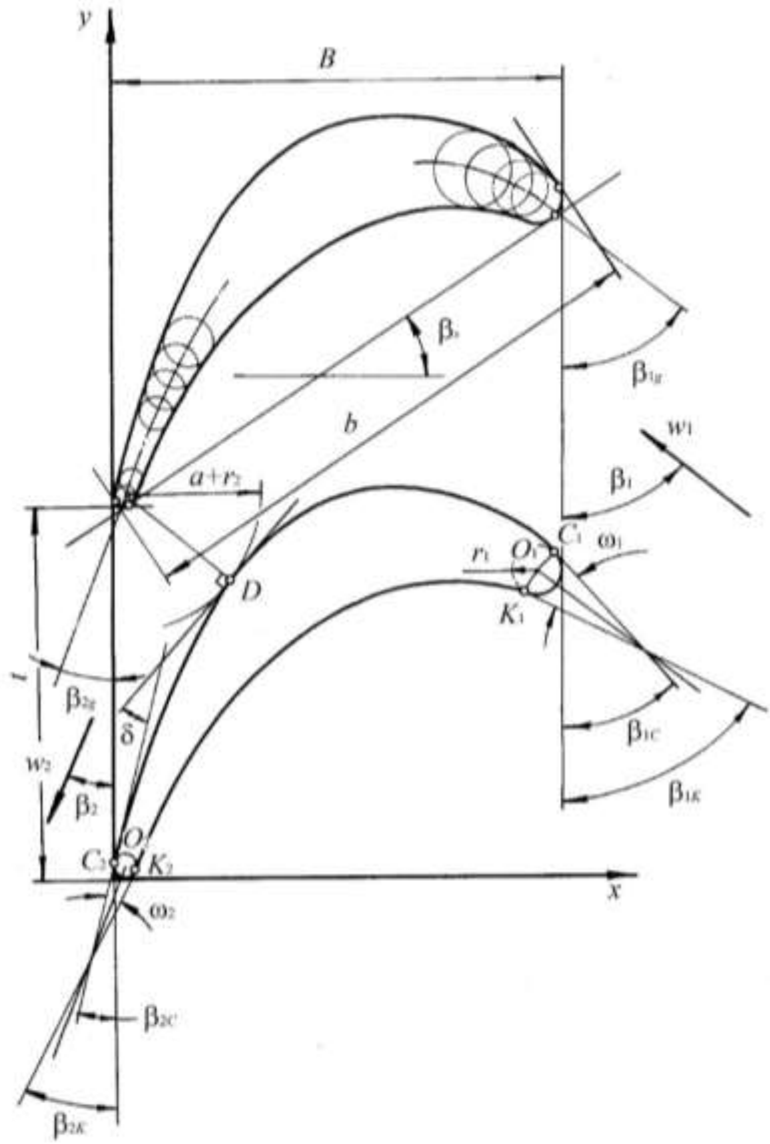


Figure 5.1 *The design parameters of the profile cascade.*

Of the last six parameters three (r_1, r_2, ω_2) are determined by calculation, the remaining three (β_s, β_{2g}, δ) can also be determined in the first approximation by the empirical formula [25]. In further at constructor's option last three parameters or part of them, may be maintained constant during the profiling, or changed, as variable parameters. As a first approximation for the profile stagger angle β_s the next relationship can be recommended:

$$\beta_s = 13.59 + 0.682\left(\beta_{1g} - \beta_{2g}\right) - 0.0028\left(\beta_{1g} - \beta_{2g}\right)^2.$$

Profile is built in a Cartesian coordinate system. Coordinates of the circle center of input and output edges, as is easily seen, is given by (Fig. 5.1):

$$\left.\begin{array}{l} x_{0_2} = r_2; \quad y_{0_2} = r_2; \\ x_{0_1} = x_{0_2} - r_1\left(\sin\beta_s + \cos\beta_s\right) + r_2\left(\sin\beta_s - \cos\beta_s\right) + b\cos\beta_s; \\ y_{0_1} = y_{0_2} - r_1\left(\sin\beta_s - \cos\beta_s\right) - r_2\left(\sin\beta_s + \cos\beta_s\right) + b\sin\beta_s. \end{array}\right\} \tag{5.1}$$

The coupling coordinates of the edges circles with convex and concave sides of the profile C_1, C_2, K_1, K_2 and their derivatives at these points are defined as follows:

$$\left.\begin{array}{l} x_{C_1} = x_{0_1} + r_1\cos\beta_{1C}; \\ y_{C_1} = y_{0_1} + r_1\sin\beta_{1C}; \\ y'_{C_1} = \mathrm{tg}\left(90^\circ - \beta_{1C}\right); \\ x_{C_2} = x_{0_2} - r_2\cos\beta_{2C}; \\ y_{C_2} = y_{0_2} + r_2\sin\beta_{2C}; \\ y'_{C_2} = \mathrm{tg}\left(90^\circ - \beta_{2C}\right); \end{array}\right\}, \tag{5.2}$$

$$\left.\begin{aligned}
x_{K_1} &= x_{0_1} - r_1\cos\beta_{1K};\\
y_{K_1} &= y_{0_1} - r_1\sin\beta_{1K};\\
y'_{K_1} &= \mathrm{tg}\left(90^\circ - \beta_{1K}\right);\\
x_{K_2} &= x_{0_2} + r_2\cos\beta_{2K};\\
y_{K_2} &= y_{0_2} - r_2\sin\beta_{2K};\\
y'_{K_2} &= \mathrm{tg}\left(90^\circ - \beta_{2K}\right),
\end{aligned}\right\} \tag{5.3}$$

Where $\beta_{1C} = \beta_{1g} - \omega_1/2; \quad \beta_{1K} = \beta_{1g} + \omega_1/2;$

$$\beta_{2C} = \beta_{2g} - \omega_2/2; \quad \beta_{2K} = \beta_{2g} + \omega_2/2.$$

The wedge angle of the leading edge ω_1 in first approximation can be determined using the guidelines [25]:

$$\omega_1 = 2.5\frac{C_{\max} - 2r_1}{b}, \tag{5.4}$$

Where $C_{\max} = 1.3f/b$.

The trailing edge wedge angle ω_2 can be set by the designer or determined by the expression:

$$\omega_2 = K_\omega \frac{0.14\omega_1}{0.2 + \omega_1}. \tag{5.5}$$

In the formulas (5.4), (5.5) the angles are in radians. The K_ω value is often taken as equal to 1. It can influence the position of the center of gravity of the profile. In the process of profile building angle ω_1 specified from the conservation of a given area.

Preserving the value of the throat *a*, for point *D* we have:

$$\left.\begin{aligned} x_D &= x_{0_2} + (a + r_2)\cos(\beta_{2C} + \delta); \\ y_D &= y_{0_2} - (a + r_2)\sin(\beta_{2C} + \delta) + t; \\ y_D &= \operatorname{ctg}(\beta_{2C} + \delta). \end{aligned}\right\} \tag{5.6}$$

In the construction of the profile convex and concave parts must first achieve coupling of describing their curves with circumferential edges, while the profile's convex part with the circumference of the throat at the point *D*. This means that these curves must satisfy the boundary conditions which are defined by formulas (5.2), (5.6) to the convex and (5.3) for the concave portions of the profile.

As for the convex part the number of these conditions is six, and for the concave – four, in order to have an opportunity to widely vary the outline profile to produce a minimum loss, the convex portion of the profile should be described by a polynomial of higher than 5-th, and the concave portion – than 3-d degree.

Let the order of the polynomial is *n*. In this case, the question of choosing the correct *n*-5 boundary conditions for the convex portion of the profile and *n*-3 boundary conditions for the concave part. As such one can take, for example, the high-order derivatives (second and higher) in the points C_2 and K_2. Not stopping until the solution of this problem, assume that the boundary conditions are somehow chosen.

Due to the fact that the number of points at which the boundary conditions are given, may be different for the convex portion and the concave profile (as mentioned above), for generality, we consider the task of determining the coefficients of the polynomial in the case of setting the boundary conditions in any number of points.

This problem is formulated as follows:

required to find the coefficients of the polynomial

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n, \tag{5.7}$$

satisfying in the k points to $n + 1$ boundary condition

$$\text{at } x = x_1:\ y = y_1,\ y' = y_1',\ \ldots,\ y^{(k_1-1)} = y_1^{(k_1-1)};$$

$$\text{at } x = x_2:\ y = y_2,\ y' = y_2',\ \ldots,\ y^{(k_2-1)} = y_2^{(k_2-1)};$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\text{at } x = x_k:\ y = y_k,\ y' = y_k',\ \ldots,\ y^{(k_k-1)} = y_k^{(k_k-1)};$$

$$\left(k_1 + k_2 + k_3 + \ldots + k_k = n + 1\right).$$

Differentiating (5.7) $\ell = \max\left\{(k_1 - 1), (k_2 - 1), \ldots, (k_k - 1)\right\}$ times by x. We assume in (5.7) and in the first $k - 1$, obtained by differentiating (5.7), the equations $x = x_1$, then (5.7) in the first and $k_2 - 1$ equations $x = x_2$, etc. until you go through all the k points at which the boundary conditions are given. Every time we get a system of algebraic equations, which for the m-th point can be written as:

$$\left.\begin{array}{l}
a_0 + x_m a_1 + x_m^2 a_2 + \ldots + x_m^n a_n = y_m; \\
a_1 + 2x_m a_2 + \ldots + n x_m^{n-1} a_n = y_m'; \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
1\cdot 2\cdot 3\cdots\left(k_m - 1\right) a_{k_m - 1} + 2\cdot 3\cdot 4\cdots k_m x_m a_{k_m} + \ldots + \\
+\left(n - k_m + 2\right)\left(n - k_m + 3\right)\ldots n x_m^{n-k_m+1} a_n = y_m^{(k_m-1)};
\end{array}\right\} \tag{5.8}$$

In the matrix form, this system can be presented as

$$C \cdot A = B,$$

where C – matrix of coefficients (5.8); A – unknown parameters column $a_0, a_1, a_2, \ldots, a_n$; B – right-hand sides of equations (5.8) column.

It is easy to see the elements of the matrix *C*, and the right-hand part column B may be determined by the following formulas:

$$\left.\begin{aligned}
&C_{i,\,j}=0,\begin{pmatrix} j=1,2,3,\ldots,k_m \\ i>j \end{pmatrix};\\
&C_{1,\,j}=x_m^{j-1},\left(j=1,2,3,\ldots,n+1\right);\\
&C_{i,\,j}=x_m^{j-1}\prod_{S=1}^{i=1}\left(j-S\right),\begin{pmatrix} j=2,3,4,\ldots,k_m \\ j=i,i+1,\ldots,n+1 \end{pmatrix};\\
&b_i=y_m^{(i-1)},\left(j=1,2,3,\ldots,k_m\right).
\end{aligned}\right\}\tag{5.9}$$

Now, if the index *m* in (5.8) will run from 1 to *k*, we arrive at a system of linear algebraic equations of order $n + 1$ relatively of unknowns $a_0, a_1, a_2, \ldots, a_n$, the elements of the coefficient matrix and the right sides of the column which are determined by formulas (5.9). Solving this system of linear equations, we will determine the coefficients of the polynomial (5.7) separately for convex and concave profile parts.

The area is calculated using the difference between the integrals of the curves describing the convex and concave portion of the profile. Be aligned with a given area can be varying wedge angle of the leading edge ω_1, repeating at the same time building a profile with the formulas (5.2), (5.3).

The developed method of turbine profiles design allows the construction of an oblique cut with straight section. Such profiles can be used for supersonic expiration and work well in conditions other than nominal.

5.2.2 Profiles Building Using Besier Curves

A more simple and clear way to build the base curve is a Bezier curve (which is especially convenient for interactive construction of complex curves), but to automate profiling with its help some special measures should be taken. There is no doubt also the fact that that the minimum of maximum curvature is a prerequisite for high aerodynamic qualities of turbine profiles cascades. In many cases, probably this criterion prevails over the condition of the absence of curvature jumps, as evidenced by still competitive CKTI profiles [33], designed from arcs and line segments.

Based on these considerations, we will build a profile consisting of two circles describing the input and output edges and three Bezier curves, one of which forms the pressure side, and the other two – convex part, respectively, from the trailing edge to the throat and from the throat to the leading edge.

Bezier curve that passes through two given end points and having at these points specified derivatives, will be called the base curve (BC).

The simplest base curve satisfying the above requirements, a Bezier curve, based on the polygon consisting of two segments passing through the given points with a given slope (Fig. 5.2). It is not difficult to assume that the use of the support polygon of the two segments gives BC, having a very large maximum curvature. In addition, when the angle between segments tends to be zero, the maximum curvature increases indefinitely.

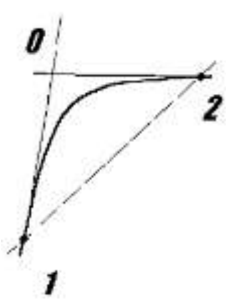


Figure 5.2 *Construction of Bezier curve by 2 points.*

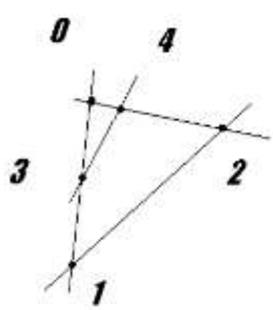


Figure 5.3 *Construction of Bezier curve in three basic segments.*

The next (and decisive) step to improving the base curve is the addition of one more segment, intersecting the first two (Fig. 5.3).

We introduce relationship

$$\frac{|1-3|}{|1-0|}=f; \quad \frac{|2-4|}{|2-0|}=g\ .$$

The course of the base curve generated by polygon *1-3-4-2* much smoother. Furthermore, it is obvious that there must be optimum values of the parameters *f* and *g*. Indeed, at *f* and *g*, aspiring to unity, we have the case of two basic segments and a very large curvature in the central part of the curve, while *f*, and *g*, tending to zero, greatly increasing curvature at points *1* and *2*.

A disadvantage of the third order base curve construction is the need to determine the optimal combination of parameters *f*, *g*, which greatly slowed the process of the profile design. Fortunately, the coefficients can be calculated only once and tabulated for different combinations of angles (Table 5.1). Since the optimum base curves do not depend on the polygon orientation or the size, the calculations can be made for the polygon, whose base is the unit interval, which lies on the *Ox* axis. In addition, due to the obvious condition

$$f_{opt}\left(\beta_1,\beta_2\right)=g_{opt}\left(\beta_2,\beta_1\right),$$

it is enough to store the data for only one optimal ratio. If you have a table of dependencies, the basic curves of sufficient quality are built almost instantly.

Table 5.1 *Optimal f and g coefficients for different angles.*

10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**Angles**
0.66	0.60	0.70	0.76	0.80	0.80	0.78	0.75	**10**
0.95	0.65	0.45	0.50	0.54	0.56	0.56	0.55	**20**
0.95	0.95	0.62	0.85	0.89	0.91	0.92	0.91	**30**
0.95	0.95	0.40	0.58	0.69	0.72	0.72	0.69	**40**
0.95	0.95	0.40	0.50	0.53	0.57	0.59	0.57	**50**
0.95	0.95	0.30	0.40	0.45	0.48	0.49	0.46	**60**
0.95	0.95	0.20	0.37	0.40	0.38	0.37	0.36	**70**
0.95	0.35	0.25	0.31	0.33	0.31	0.27	0.20	**80**

Profile is constructed from two circles that form the input and output edges, one of BC, which describes the pressure side, and the two BCs, describing the suction side. In this way, initial profiling parameters are listed in Section 5.2.1 (Fig. 5.1).

This information is sufficient to build the support polygons of the profile sections. Formulas for determining the coordinates of the corresponding points and angles do not differ from those given in the previous section. An algorithm for constructing the profile is very simple, but it has a major disadvantage: in the point of the throat, where two base curves are joined, it is possible discontinuity of curvature, which may lead to local deformation of profile velocity, and a sharp increase in the friction loss. There is a simple way to smooth BC docking at the throat. It lies in the selection of the unguided turning angle to match the curvature of parts at the throat point. Because of the high curvature sensitivity of the unguided turning angle, the variation turns minor. Determination of δ_{opt} is carried out by solving the equation

$$\kappa_{1D}(\delta) = \kappa_{2D}(\delta)$$

by secant method.

Elimination of the curvature jump in the throat requires only a few profile evolutions and a decision is reached very quickly. Built in such a way will be called the basic profile (BP). After a slight modification the algorithm also allows to construct suitable profiles with elongated front part.

It should be borne in mind that the BP is not yet the final product, it is only a semifinished product intended for optimization of all the others, except for the initial, data. This optimization can be performed according to different criteria.

In the process of BP constructing assumed the specified parameters with the exception of the unguided turning angle, which was chosen in such a way as to eliminate the curvature jump at the throat. The remaining ten parameters can be varied to optimize a chosen cascade optimality criterion.

In general, the problem of optimal design of a flat cascade can be written as:

$$\min F(\mathbf{X}), \quad \mathbf{X} \in \Omega_X . \tag{5.10}$$

Vector of variable parameters X should in some way describe the shape of the profile. Criterion $F(X)$ is a functional on X. Restrictions on the range of admissible values of the vector X associated with strength and technological requirements cascade imposed on, which are, in particular, the shape and thickness of the input and output edges. Because of the sufficient simplicity of accepted method for calculating the tensile and bending stress in the blade section, they can be defined directly in the process of the profile shape optimization. However, we will stick to a different approach, considering approximately known basic cascade dimensions (chord, relative pitch, etc.) on the basis of the calculation described in section 5.1.

Specifically, a vector of variable parameters includes the following characteristics which influence the configuration of the profile that is based on the procedure described in the previous section:

- profile stagger angle;
- relative pitch;
- geometrical exit angle;
- the radius of the leading edge;
- wedge angle of the leading edge;
- wedge angle of the trailing edge.

Restrictions on the range of the parameter is written in the simplest form:

$$\bar{X}_{\min} < \bar{X} < \bar{X}_{\max} . \tag{5.11}$$

If you wish to fix a component X_i we believe $X_{i\min} = X_{i\max}$.

The most important point in the cascade optimization is the correct criterion of quality selection, which generally represents the minimum total loss of kinetic energy in the cascade taking into account the relative time of its operation at different flow regimes in a given stage of the turbine. In connection with this problem distinguish multi-mode and single-mode optimization solution requires the calculation of cascade flow and constituting losses therein, respectively, at set of modes or in one of them.

As shown by previous studies, in some cases, an alternative criterion of aerodynamic quality can be geometric criterion of the profile smoothness. One could even argue that this observation even more relevant to a multi-mode optimization, than single-mode. The original method was developed in relation to the profiles submitted by power polynomials.

5.3 Optimization of Geometric Quality Criteria

When used for the formation of the profile contour of polynomials of degree n ($n > 5$ for the convex part of the profile, and $n > 3$ for the concave part) the question arises about the correct choice of the missing n–5 (or n–3) boundary conditions which must be selected on the basis of the requirements of aerodynamic profile perfection.

One of the requirements of building the turbine profiles with good aerodynamic qualities is a gradually changing curvature along the outline of the profile [25]. Unfortunately, the question concerning the nature of the change of curvature along the profile’s surface, is currently not fully understood.

As a geometric criterion for smooth change of curvature in the lowest range of change in the absence of kinks on the profile, you can take the value of the maximum curvature on the profile contour in the range $\left[x_{C_2}, x_{C_1}\right]$ for the convex and for $\left[x_{K_2}, x_{K_1}\right]$ the concave parts, by selecting the minimum of all possible values at the profile designs with the accepted parameters and restrictions. The requirement for the absence of curvature jumps in the description of the profile contour by power polynomials automatically fulfilled as all the derivatives of the polynomial are continuous functions. Agree to consider determined based on the geometric quality criterion, the missing boundary conditions in the form of derivatives of high orders in points C_2, and K_2 components of a vector $\vec{Y}$. For the concave part of the profile vector of varied parameters $\vec{Y}$ is as follows:

$$\vec{Y}_K = \left\{y''_{K_2}, y'''_{K_2}, \ldots, y^{(n-3)}_{K_2}\right\}.$$

For the convex part to the derivatives of high orders added geometrical exit angle β_{2g} and at constructor's option unguided turning angle δ:

$$\vec{Y}_C = \left\{y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-5)}_{C_2}, \beta_{2g}, \delta\right\}.$$

To construct the optimal profile is taken such a vector $\vec{Y}_{opt}$, which provides the minimum of the functional

$$F\left(\vec{Y}\right) = \max\left(k\right), \tag{5.12}$$

wherein k – the curvature of the profile, and the maximum is searched for in the range $\left[x_{C_2}, x_{C_1}\right]$ on the convex portion of the profile and $\left[x_{K_2}, x_{K_1}\right]$ – on the concave part of the profile using one of the one-dimensional search methods.

Formulated the problem of minimizing the functional (5.12) can be solved by the methods of nonlinear programming. In this case, a very successful was a flexible polyhedron climbing algorithm.

An algorithm for an optimal profile constructing using the geometric quality criterion is as follows:

1. For the given values of the inlet and outlet edge radii r_1 and r_2, chord *b*, received or estimated using particular one of the recommended dependencies profile's stagger angle β_s, the coordinates of the centers of inlet and outlet edges circles calculated using (5.1).

2. Set the leading edge wedge angle ω_1.

3. Select the initial approximation for β_{2g}, δ, by which and adopted value of ω_2 by the formulas (5.2), (5.6) the coordinates of the points C_1, C_2 and *D* are determined, as well as their first derivatives.

4. Determine the coefficients of the polynomial (5.7), which describes the convex portion of the profile. Wherein high order derivatives $y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-5)}_{C_2}$ are set as the initial approximation in the first step and refined during the optimization.

5. By using one of the methods of one-dimensional search the maximum curvature $\max|k|$ objective function value is found. Next on the program for searching the extremes minimum of the functional sought

$$F\left(\vec{Y}\right) = \max|k|.$$

Minimum of the functional corresponds to the optimal value of the vector of varied parameters $\vec{Y}_{C_{opt}} = \left\{y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-5)}_{C_2}, \beta_{2g}, \delta\right\}$ by which at this stage of the profile building the coefficients of the polynomial (5.7) describing the profile's convex part are determined.

6. From formulas (5.3) calculates the coordinates of the points K_1, K_2, and their first derivatives. Varying the vector $\vec{Y}_K = \left\{ y''_{K_2}, y'''_{K_2}, \ldots, y^{(n-3)}_{K_2} \right\}$ by the means of optimization program a value $\vec{Y}_{K_{opt}}$ and the coefficients of the polynomial that describes the profile concave portion are searched.

7. Determine the area of the profile $f(\omega_1)$ and the discrepancy $F = \left| f(\omega_1) - f \right|$. Setting a new ω_1 value, profiling process is performed again from step 3. As stated above, the minimum residual is achieved by using the "golden section" one-dimensional search of extremum procedure.

It was also developed somewhat different algorithm for constructing an optimal profile of the geometric quality criteria.

The main stages of the algorithm are as follows:

1. Setting a constructive exit angle $\beta_{2g} = \arcsin a/t$, define as a first approximation, the profile stagger angle in the cascade by the recommended [25] formula

$$\mathrm{tg}\,\beta_s = 0.2 + 0.8\left(\beta_{1g} - \beta_{2g}\right),$$

and the coordinates of the centers of inlet and outlet edges circles O_1 and O_2, by using the specified values of the radii r_1 and r_2, the chord *b*.

2. Calculating from the formula (5.4), (5.5) the edges wedge angles ω_1 and ω_2, determine the coordinates of the points of contact C_1 and C_2 on the convex side, K_1 and K_2 on the concave part of the profile, as well as the first derivatives in them.

3. We determine the coefficients of the polynomial (5.7), which describes the convex portion of the profile. The derivatives of higher orders $y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-3)}_{C_2}$, necessary to determine the coefficients, are set as the initial approximation in the first step and refined during the optimization.

4. Using the method of one-dimensional search of extremum – the "golden section" method in the range x_{C_2}, x_{C_1} the value of maximum curvature $\max|k|$ is found, the minimum of which is determined by the Nelder-Mead nonlinear programming method, changing the vector of varied parameters $Y_C = \left\{y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-3)}_{C_2}\right\}$.

5. Constructing the convex portion of the profile, drop the perpendicular from the center of the circle O_2 of the neighboring cascade profile trailing edge and determine the size of inter-blade channel throat $O_2D - r_2$. Having the difference between the obtained value and the predetermined throat

$$\Delta a = \frac{x_D - x_{O_2}}{y'_D}\sqrt{1+\left[y'_D\right]^2} - \left(a + r_2\right),$$

refine by the recommendations [25], the profile stall angle β_s and constructive exit angle β_{2g}:

$$\operatorname{tg}\beta_s\left(i+1\right) = \frac{\operatorname{tg}\beta_{si} + \Delta\beta_{si}}{1 - \operatorname{tg}\beta_{si}\Delta\beta_{si}}; \quad \beta_{2g}\left(i+1\right) = \beta_{2gi} - \Delta\beta_{si},$$

where

$$\Delta\beta_{si} = \frac{\Delta a}{2\sqrt{\left(x_D - x_{O_2}\right)^2 + y_D^2}}.$$

The process of the profile convex portion constructing continue from step 1 until the throat is not held with the desired accuracy.

1. Varying the vector $\vec{Y}_K = \left\{ y''_{K_2}, y'''_{K_2}, \ldots, y^{(n-3)}_{K_2} \right\}$, using optimization in the range x_{K_2}, x_{K_1} the values of $\vec{Y}_{K_{opt}}$ are sought as well as the coefficients of the polynomial, describing the concave part of the profile (points 3, 4).

2. Determined the profile area $f\left(\omega_1\right)$ and the discrepancy $F = \left| f\left(\omega_1\right) - f \right|$. Given a new value ω_1 and profiling process is carried out again from step 2. Minimization of F residual is achieved by using an one-dimensional search of extreme.

3. Using one of possible methods, profile velocity distribution and boundary layer are calculated. Profile quality control is carried out by the nature of the velocity distribution around its contours, the value of profile loss and the boundary layer separation criteria.

The calculation of the velocity distribution around a plane cascade profile and loss coefficients made by sequentially the following tasks: calculation of potential ideal incompressible fluid flow around a flat cascade; approximate calculation of the compressibility of the working fluid; the boundary layer calculation and loss factor determination.

Methods for potential flow of an incompressible ideal fluid calculation in the plane cascade can be divided into methods based on conformal mapping of the flow domain and methods of solving tasks given to integral equations [8, 22].

Considering the profile loss ratio ζ_{pr} as the sum of the friction ζ_{fr} and edge losses ζ_e coefficients using proposed in [8] approximate formula for determining the value of the expression ζ_{pr} can be written as:

$$\zeta_{pr} = 2\frac{\delta_{ss}^{**} + \delta_{ps}^{**}}{t\sin\beta_2} + 0.1\frac{2r_2}{t\sin\beta_2}, \tag{5.12}$$

wherein $\delta_{ss}^{**}, \delta_{ps}^{**}$ – the momentum thickness on the convex (suction side) and the concave (pressure side) portions of profile.

The calculation of the boundary layer can be produced by known methods of boundary layer theory [22]. There is reason to believe the boundary layer in real turbomachinery cascades fully turbulent. At least the treatment the boundary layer as turbulence do not gives low loss coefficient values in the cascades. Before values of Mach numbers M < 0.5, calculation of the boundary layer on a single cascade profile can produce satisfactory accuracy as an incompressible fluid [22]. As a possible formulas for the momentum thickness calculation can take the expression obtained in the solution of the turbulent boundary layer by L.G. Loytsyanskiy method

$$\delta^{**} = 0.0159\,\mathrm{Re}^{-0.15}\, w_2^{-3.55}\left(\int_0^S w^4 dS\right)^{0.85}, \tag{5.14}$$

where Re – Reynolds number; w_2 – cascade output velocity; $w(S)$ – the profile countour velocity distribution function.

The integral in (5.14) is determined by a numerical method. Determined with the help of (5.14) the $\delta_{ss}^{**}, \delta_{ps}^{**}$ values, and substituting them into (5.12), we will find the profile loss ratio.

5.4 Minimum Profile Loss Optimization

A more rigorous formulation of creating an optimal cascade profile problem that provides design parameters of the flow at the exit and meet the requirements of strength and workability, is the problem of profiling, which objective function is the profile (or even better – integral) losses.

As mentioned above, the profile loss ratio can be presented as the sum of the friction loss coefficients of the profile ζ_{fr} and edge loss coefficient ζ_e.

Given that the ratio of the edge losses associated with the finite thickness of trailing edges, the value of which is predetermined and is practically independent of the profile configuration, the objective function can be assumed as [8]

$$\zeta_{fr} = 2\frac{\delta_{ss}^{**} + \delta_{ps}^{**}}{t\sin\beta_2}. \tag{5.15}$$

In terms of flow profile, you must set a limit, excluding the boundary layer separation. Unseparated flow conditions according to Buri criterion can be written as [22]:

$$-\frac{\delta^{**}}{w}\frac{dw}{dS} \le B\left(\mathrm{Re}^{**}\right)^{-\frac{1}{m}}, \tag{5.16}$$

Where $\mathrm{Re}^{**} = \mathrm{Re}\,\delta^{**}/b$.

The constants B and m can be taken equal to: $B = 0.013\ldots0.020$, $m = 6$.

The task is set of determining the coefficients of the polynomials (5.7) for a description of the convex and concave profile with given geometric, strength and processability parameters so as to reach the minimum of the functional (5.15) and satisfy the constraints (5.16).

Formulated the optimal profiling problem is essentially non-linear with inequality constraints and mathematically formulated as follows:

$$\min f\left(\vec{Y}\right), \quad g\left(\vec{Y}\right) \ge 0, \tag{5.17}$$

where $\vec{Y}=\left\{\beta_s, \beta_{2g}, \delta, \beta_{1g}, y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-5)}_{C_2}, y_{K_2}, y'''_{K_2}, \ldots, y^{(n-3)}_{K_2}\right\}$ vector of varied parameters objective function, whose role in the problem plays an equation for the coefficient of friction (5.12); $g\left(\vec{Y}\right)$ – constraint, which on the basis of Buri separation criterion (5.14), is defined as follows:

$$g\left(\vec{Y}\right)=\max g_i\left(\vec{Y}\right); \tag{5.18}$$

$$g_i\left(\vec{Y}\right)=\begin{cases} G_i, \text{ at } G_i>0; \\ 0, \text{ at } G_i<0, \end{cases}$$

where

$$G_i=B\left(\mathrm{Re}^{**}\right)^{-\frac{1}{m}}+\frac{\delta_i^{**}}{w_i}\left(\frac{dw}{dS}\right)_i, \tag{5.19}$$

i = 0, 1, ..., 2n (2n – the number of points on the profile contour).

Applying to the problem solution method the penalty functions method [3], we reduce the problem of finding the extremum in the presence of constraints to the problem without restriction. Form the generalized functional I^*

$$I^*=\zeta_{fr}+\Lambda g\left(\vec{Y}\right), \tag{5.20}$$

where Λ – penalty coefficient.

For the unconstrained minimization of the functional (5.20) Nelder and Mead algorithm was used [3].

An algorithm for constructing an optimal profile of the minimum profile loss is as follows:

1. As the initial data for profiling on the basis of thermal calculation and the conditions of durability and adaptability the quantities are introduced:

a – throat inter-blade channel; b – chord; t – cascade step; f – profile square; r_1 and r_2 – input and output edges radii; ω_2 – trailing edge wedge angle.

2. An initial approximation for the leading edge wedge angle ω_1, the stagger angle of the profile β_s, geometric (constructive) entry β_{1g} and exit β_{2g} angles, unguided turning angle δ, derivatives of higher orders $y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-5)}_{C_2}, y_{K_2}, y'''_{K_2}, \ldots, y^{(n-3)}_{K_2}$.

3. Determines the coordinates of the points C_1, C_2, D, K_1, K_2, and their first derivatives.

4. Sought the coefficients of polynomials describing the concave and convex portion of the profile according to the procedure set out in section 5.1.

5. The profile area determined and, using one of the one-dimensional search methods, varying angle ω_1, a minimum of residual $F = \left| f(\omega_1) - f \right|$ is found. The process of profiling is carried out from step 2.

6. Calculate the profile velocity distribution, as well as the coefficient of friction ζ_{fr} by (5.15) and the G_i value by (5.19).

7. We call the routine of optimization for finding the minimum of the functional (5.20), each time making the profile area fit before the calculation of the objective function. A minimum of the functional (5.20) corresponds to the optimum value of the vector of variable parameters

$$\vec{Y}_{opt} = \left\{ \beta_s, \beta_{2g}, \delta, \beta_{1g}, y''_{C_2}, y'''_{C_2}, \ldots, y^{(n-5)}_{C_2}, y_{K_2}, y'''_{K_2}, \ldots, y^{(n-3)}_{K_2} \right\}.$$

8. The optimal profile construction is made, satisfying the strength, geometrical and technological constraints, and provides a minimum profile loss while maintaining the unseparated flow. By the designer's wish optimization

may also be performed using the parameter t/b, and the trailing edge wedge angle ω_2.

5.5 Optimal Profiling Examples

The created profiling algorithms have allowed to design a series of profiles of turbine cascades.

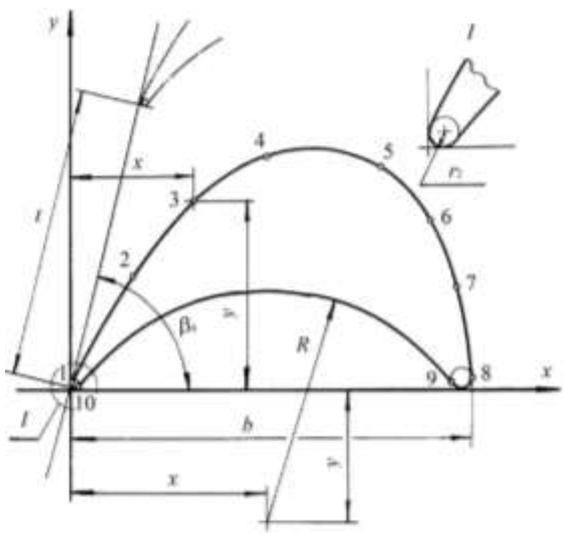
As a starting (*1O*) was taken the standard profile *P2* with a high aerodynamic quality. Wherein were accepted such flow conditions that ensure the smallest possible profile *P2* (*1O*) losses: $\bar{t} = t/b = 0.722$, β_b = 76°26′, β_1 = 29°30′. Retaining the basic, necessary for the machine profiling raw data $(\bar{t}, \beta_b, b, a, f, r_1, r_2, \beta_1)$ with the help of the developed algorithms were obtained new profiles: *1MMC* (for the geometric quality criteria – the minimum of maximum curvature) and *1MPL* (the minimum of profile loss).

From technological considerations subsequently profile *1MMC* contour was approximated by the radii (Fig. 5.4, 5.5, Table 5.2). Fig. 5.6–5.8 shows the distribution of the velocity and the parameter *B* (the Buri boundary layer separation criterion) along the contours of the original and newly created profiles.

The calculated profile loss ζ_{pr} values correspondingly are 3.35, 3.16 and 3.00%. Attention is drawn to the different law of the parameter *B* variation along the profiles contours. Apparently, the possibility of the boundary layer separation, or the intensity of its thickening (which leads to increased losses) must be judged not only by the maximum value of the parameter *B*, which (usually) achieved at cascade’s oblique cut, but also the character of its change within the channel prior bevel, particularly on the convex side of the profile.

For comparative testing of profiles *1O*, *1MMC* and *1MPL* were chosen conditions of the original flow profile *P2* (*1O*) which provide the smallest possible losses: $\bar{t} = t/b = 0.722$, $\beta_b = 76°26'$.

All nominal dimensions of the experimental blades and cascades of considered profiles adopted respectively the same, namely a chord b = 42 mm; length of blade l = 120 mm; pitch t = 30.32 mm; channel throat a = 10.85 mm; the thickness of the trailing edge δ = 0.66 mm. The stagger angles for the newly designed profiles *1MMK* and *1MPP* equaled stagger angle of the source profile *1O*.

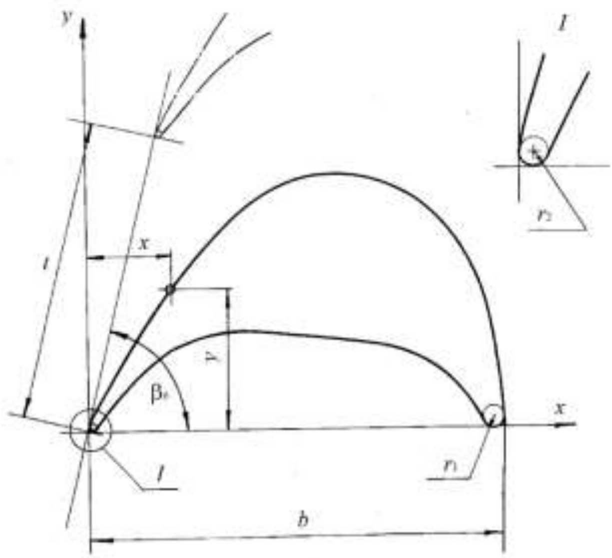


β_b °	76°26′	Bilateral points	Coordinates, mm		Bilateral points	Coordinates, mm	
b, mm	420,0		*x*	*y*		*x*	*y*
δ_{in}	6,66	1	0,3334	4,7837	6	377,3304	174,9175
δ_{out}	$1{,}59 \cdot 10^{-2}$	2	64,5584	118,8692	7	404,2919	104,6999
C, mm	153,6276	3	128,4423	196,9085	8	419,9	12,7715
t, mm	303,240	4	204,5439	244,4601	9	399,1575	4,6840
f, sm^2	479,962	5	324,6853	230,7689	10	6,0428	1,3995

Figure 5.4 *Profile 1MMC.*

Table 5.2 *1MMC profile parameters in the radiusographic form.*

Arcs	Radii and centers coordinates of arcs, mm			Arcs	Radii and centers coordinates of arcs, mm		
	R	***x***	***y***		***R***	***x***	***y***
1-2	1057,0	951,6247	–455,8897	6-7	277,5	134,1478	41,244
2-3	455,0	446,4076	–128,5415	7-8	750,0	–325,8325	–66,7955
3-4	210,0	275,1959	46,7023	8-9	11,55	408,4469	11,5460
4-5	137,7	250,8724	114,7901	9-10	242,99	203,8004	–139,8127
5-6	155,0	241,50	100,2534	10-1	3,33	3,33	3,33



β_b °	76°26′
b, mm	420,0
δ_{out}, mm	6,66
δ_{in}, mm	$1{,}59 \cdot 10^{-2}$
C, mm	164
t, mm	303,240
f, sm^2	479,0

X_{O_2}, mm	3,33
Y_{O_2}, mm	3,33
r_2, mm	3,33
X_{O_1}, mm	408,451
Y_{O_1}, mm	11,55
r_1, mm	11,55

Figure 5.5 *Profile 1MPL.*

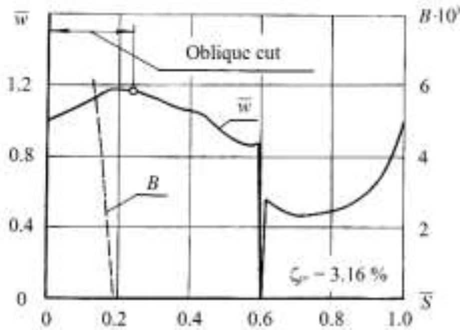


Figure 5.6 Distribution of velocity and parameter B along the 1MMC profile contour. Calculated profile loss coefficient $\zeta_{pr} = 3.16\%$.

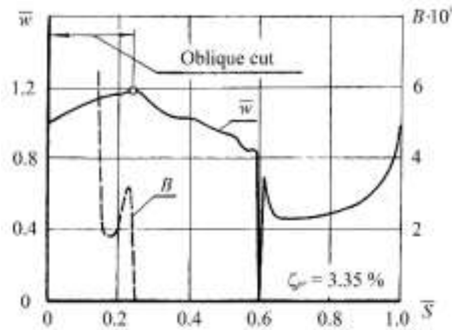


Figure 5.7 Distribution of velocity and Buri separation criteria B along the 1MPL profile contour. Calculated profile loss coefficient $\zeta_{pr} = 3.35\%$.

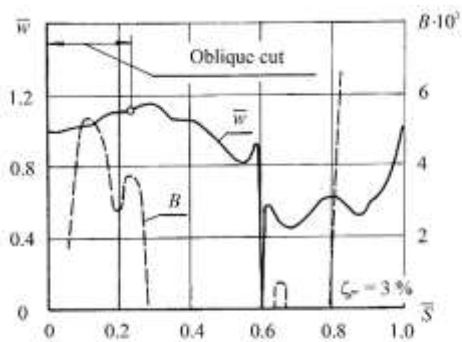


Figure 5.8 Distribution of velocity and parameter B along the 1MPL profile contour. Calculated profile loss coefficient $\zeta_{pr} = 3.0\%$.

In the blades manufacture the profile was controlled by the working patterns. The template fit appears on the projector using the drawing profile contour 10 times increased relatively to the blade profile. When fit the profile by template contour clearance allowed not more than 0.04 mm. It should be noted that the difference in the contours of the most similar profiles *1O* and *1MMC* reaches 0.6 mm, i.e. an order of magnitude greater of the blades manufacture tolerance. Particular attention was paid to ensure a predetermined trailing edge thickness. The admission to the size of the throat when building cascades was ±0.03 mm. Effective angle downstream was $\beta_{2e} = \arcsin a/t =$ 20°55′.

The aim of the tests was to obtain comparative data on the profile losses factors ζ_{pr} and exit flow angles β_2 in the subsonic region, at the range of Mach numbers 0.3…0.65, and different inlet flow angles β_1.

The comparability of the experimental results was ensured by making the blades and cascades in the same manner with the same requirements for precision and surface finish; cascade tests one the same test rig, using the same instrumentation and the measured data processing methods.

The main test was preceded by methodological tests. On expiration mode of Mach M_{2T} = 0.46, the measurements were carried out along the front of the cascade at different distances from the plane of output edges and in the three sections of the height of the blades. The values of certain kinetic energy loss factor ζ_{pr} is calculated for the measurement intervals along the cascade front multiple of two, three and four steps of the blades. The results of such averages practically coincided, indicating that careful manufacture of blades and high quality cascades assembly.

As a result of preliminary tests it was found that the averaged energy losses in the flow behind cascade will stabilize at a distance from the 0.25*b* of the trailing edges. Thus for a layer thickness of 20% of the blades height, symmetrical about their middle, the flow is very close to the flat.

Final testing data of three experimental cascades were obtained by measurements on the middle section of the height of the blades at a distance equal to 0.285*b* from the trailing edges in the three-step interval.

Fig. 5.9 shows the experimental dependences of the cascade profile losses versus inlet flow angle β_1 in the range of change from 26° to 41° at different Mach numbers ranging from 0.45 to 0.68, which corresponds to Reynolds numbers of Re = $3.9 \cdot 10^5$ to Re = $5.75 \cdot 10^5$. In these intervals profile losses curves of cascade made up of the original profile *1O*, are located above the profile losses curve of newly designed cascade *1MMC*. Both profiles have minimum profile loss at inlet flow angle β_1 = 35°. The magnitude of profile loss in the second cascade of 0.3…0.4% less than the first substantially throughout the whole range of variation of the input flow angle β_1 in the specified range of the Mach number values.

Wherein loss in each of the cascades *1O* and *1MMC* increasing against the minimum value of 0.8% in the case of 5° deviation of input flow from the optimum angle β_1 = 35°. The minimum profile losses amount of the cascade, composed from the newly designed blades *1MMC*, optimized for geometric quality criterion, is 2.2 %.

Profile losses of cascade, composed of profiles *1MPL*, were slightly lower of cascades *1O* and *1MMC* at the nominal input flow angle β_1 = 29°30′. With the inlet flow angle decreasing, *1MPL* profile advantage slightly increases. However, at the inlet flow angles β_1 > 30° profile *1MPL* is worse than others. It should be emphasized that this profile losses factor curve vs input flow angle β_1 in the investigated range of Mach numbers has a minimum at the angle β_1 = 29°30′, under which the profile *1MPL* was designed.

Fig. 5.10 shows the dependence of the angles downstream the cascades β_2 of the input flow angle β_1 at different Mach numbers. The newly designed cascade 1MMC has the better match of the output flow angle β_2 with the effective angle $\beta_{2eff} = \arcsin a/t$ value in the entire tested range. A similar pattern is observed for the cascade of *1MPL* profiles within its region of advantages.

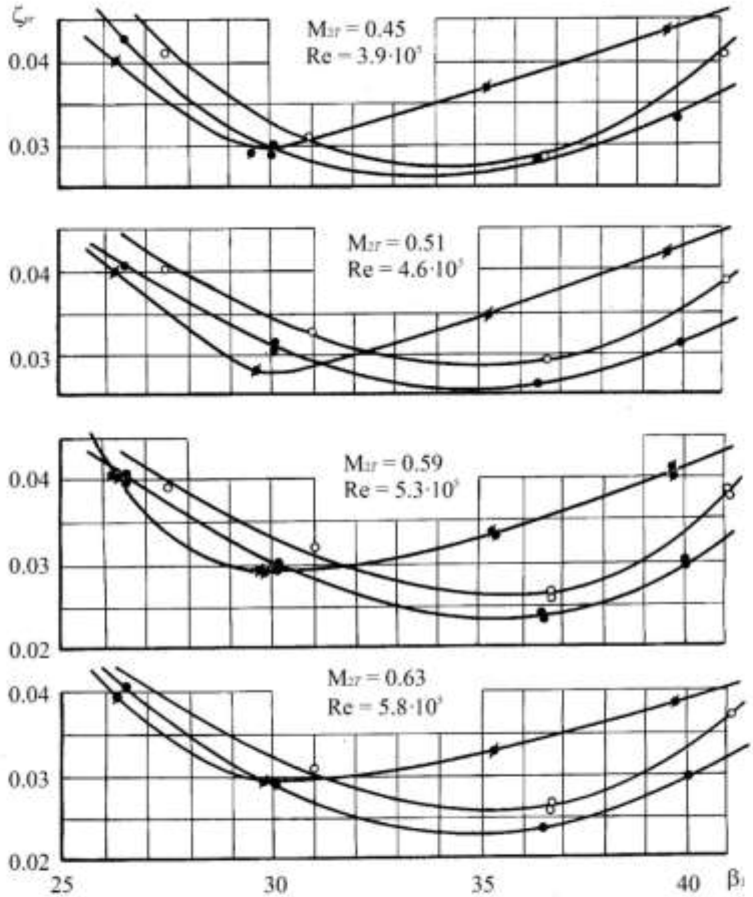


Figure 5.9 *Test results of cascades 1O (—o—), 1MMC (—● —) and 1MPL (—▪—). Test conditions: b = 42 mm; t/b = 0.722; l/b = 2.857; a = 10.87 mm;* $\bar{\delta}$ *= 1.59·10*$^{-2}$*;* β_b *= 76°26′.*

The another results of optimal profiling of cascades with converging and diffuser channels, as well as data of their experimental studies, can be found in [13].

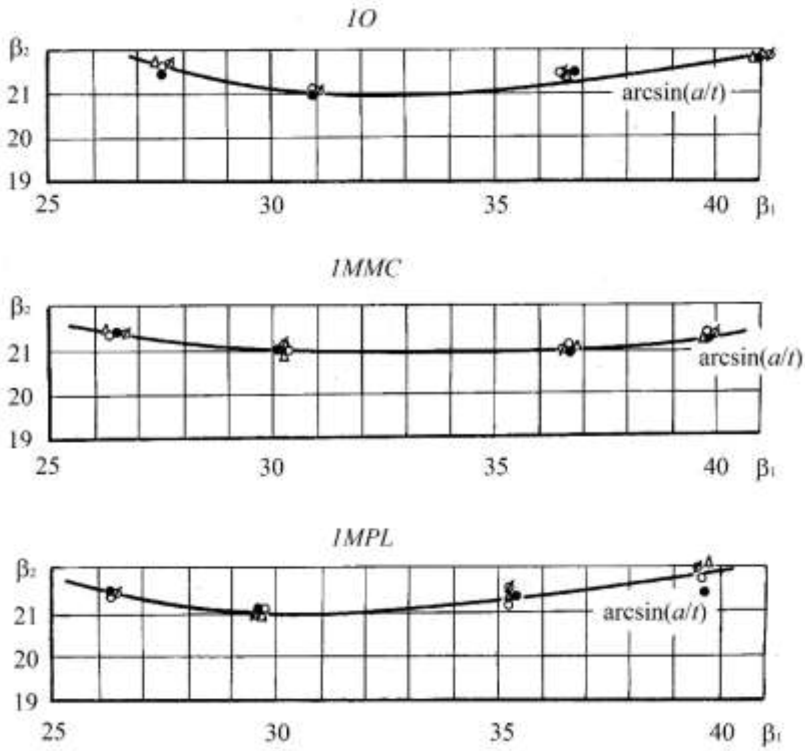


Figure 5.10 *Test results of cascades 1O, 1MMC and 1MPL at different Mach numbers:* $\Delta -\ M_{2T} = 0.37$; $\varnothing -\ M_{2T} = 0.45$; $O -\ M_{2T} = 0.51$; $\bullet\ -.\ M_{2T} = 0.59$.

The results obtained to build the turbine cascades of a minimum profile loss authenticate the proposed statement of the profiling problem. Of course, for such problems more correct to take as an objective function the integral loss, what is the most naturally achieved involving computational aerodynamics models.